Circular Motion Problems – ANSWERS

1. An 8.0 g cork is swung in a horizontal circle with a radius of 35 cm. It makes 30 revolutions in 12 seconds. What is the tension in the string? (Assume the string is nearly horizontal)

   | T=time/revolutions=0.4 s | Period is the time per revolution |
---|---|---|
ΣF=ma | Write down N2L |
F_{tension} = \frac{mv^2}{r} | Tension provides net force, acceleration is centripetal |
F_{tension}=m(4\pi^2r/T^2) | Speed equals circumference divided by period |
F_{tension}=1.7 N | Substitute values and calculate |

2. A 15 g stopper is swung in a horizontal circle with a radius of 0.80 meters. The tension in the string is 1.5 Newtons. Find the speed of the stopper and determine how long it takes to complete 30 revolutions. (Assume the string is very nearly horizontal).

   | ΣF=ma | Write down N2L |
---|---|---|
F_{tension} = \frac{mv^2}{r} | Tension provides net force, acceleration is centripetal |
v=sqrt(F_{tension} r / m) = 8.9 m/s | Solve for v and calculate |
v=2\pi r / T \Rightarrow T=2\pi r / v = 0.56 s | Speed equals circumference divided by period |
t = 30 T = 16.9 s | Time = # rev x seconds/rev |

3. A brass ball with a mass of 120 grams is suspended from a string that is 60.0 cm long. The ball is given a push and it moves in a horizontal circle. The string is not nearly horizontal. It forms an angle of just 22.6 degrees from the vertical. (This is sometimes called a conical pendulum because the string sweeps out the surface of a cone).

   a. Draw a free body diagram indicating the forces acting on the ball.
   b. What is the y-component of the tension force equal to? How do you know?
   c. Use trigonometry to find the x-component of the tension force.
   d. What is the radius of the ball’s motion?
   e. Use your answers to c & d to find the speed of the ball.

   | ΣF_y=0 | Motion is horizontal so y-forces must cancel |
---|---|---|
y-component = 1.2 N | y-component cancels force of gravity |
\tan 22.6° = F_{Tx}/F_{Ty} | SOH-CAH-TOA |
F_{Tx}=1.2 \tan 22.6 = 0.50 N | Solving for x-component |
r=0.60 \sin 22.6 = 0.231 | The radius of the horizontal circle is the distance from the ball to the vertical axis |
ΣF_x=mv^2/r | Horizontal force provides centripetal acceleration |
v=sqrt(F_{Tx} r / m) = 0.98 m/s | Solve for v |
4. A 1200 kg car drives at a constant speed of 14 m/s around a circular track (r=80.0m).
   a. What is the size of the net force acting on the car?
   b. What is the physical agent providing that force?
   c. What is the maximum frictional force that can act on the tires if the static coefficient of friction is 0.30? Will the car’s tires start slide? If not, how fast can the car move before it does start sliding?

   \[
   \text{F}_{\text{net}} = ma = \frac{mv^2}{r} = 2940 \text{ N}
   \]
   Car experiences a centripetal acceleration

   Friction provides the force
   Without friction the car could not turn!

   \[F_s < \mu_s F_N \Rightarrow F_{\text{max}} = 3600 \text{ N} \]
   The normal force is balancing gravity pulling down

   At 14 m/s the car won’t slide
   The force needed (2940 N) is less than max friction

   \[
   \mu_s mg = \frac{mv^2}{r}
   \]
   Set max frictional force equal to m a \text{centripetal}

   \[
   v = \sqrt{\mu_s gr} = 15.5 \text{ m/s}
   \]
   Solve for v

5. How fast can a car travel around an unbanked curve with a radius of 60 meters if the coefficient of friction is 0.40?

   Using the result from above \(v = \sqrt{0.4 \times 10 \times 60} = 15.5 \text{ m/s}\) (higher friction but smaller radius happens to result in the same maximum speed)

6. What is the minimum coefficient friction necessary to keep a car from sliding if it travels around an unbanked curve (r=140 meters) at a speed 25 m/s?

   Using the result from #4 and rearranging: \(\mu_s = \frac{v^2}{gr} = 0.45\)
7. Tarzan (m=90 kg) swings from a vine that is 3.0 meters long. His speed at the bottom of the swing is 4.0 m/s.
   a. What is his centripetal acceleration?
   b. What is the NET force acting on him?
   c. What is the tension in the vine?

   \[ a_c = \frac{v^2}{r} = 5.33 \, \text{m/s}^2 \]

   \[ \Sigma F = ma = 480 \, \text{N} \]

   \[ F_{\text{tension}} - F_{\text{gravity}} = ma \]

   \[ F_{\text{tension}} = 480 + 900 = 1380 \, \text{N} \]

   (tension has to cancel his weight PLUS provide an inward force to deflect him from his current direction of motion)

8. A hot wheels track has a vertical loop with a radius of 20 cm.
   a. What is the minimum speed the car can have at the highest point without falling off of the track?

   \[ \Sigma F = ma \]

   \[ F_{\text{gravity}} + F_{\text{normal}} = \frac{mv^2}{r} \]

   \[ mg = \frac{mv^2}{r} \]

   (at minimum speed \( F_{\text{normal}} \) drops to zero)

   \[ v = \sqrt{gr} = 1.4 \, \text{m/s} \]

   b. If the actual speed is 1.8 m/s, what is the normal force? (use m=20 grams)

   \[ F_{\text{gravity}} + F_{\text{normal}} = \frac{mv^2}{r} \]

   \[ F_{\text{normal}} = \frac{mv^2}{r} - mg = 0.124 \, \text{N} \]

9. An 800 kg car goes over a hill. At the top of the hill the radius of curvature is 24 meters.
   a. If the car is traveling at a speed of 12 m/s what is the NET force needed to keep the car following the curve of the hill? \( \Sigma F = ma = \frac{mv^2}{r} = 4800 \, \text{N} \)

   b. What is the normal force acting on the car as it goes over the top at this speed?

   \[ F_{\text{grav}} - F_{\text{norm}} = ma \rightarrow F_{\text{norm}} = F_{\text{grav}} - ma = 3200 \, \text{N} \]

   c. What is the maximum speed at which a car could drive over this hill without going airborne? Normal force drops to zero at max speed, \( mg = \frac{mv^2}{r} \rightarrow v = \sqrt{gr} = 15.5 \, \text{m/s} \)
10. It takes Mars 1.9 years to complete an orbit around the sun. Use Kepler’s third law to determine the average distance between Mars and the Sun. Give your answer in AU’s (astronomical units).

\[ \frac{T_e^2}{r_e^3} = \frac{T_M^2}{R_M^3} \]

\[ R_M = R_e \left( \frac{T_M}{T_e} \right)^{2/3} = (1.92)^{2/3} = 1.5 \text{ AU} \]

11. Mars has two moons Phobos, and Deimos. Use the data given to test if these moons obey Kepler’s third law of planetary motion.

Phobos: \( r = 9400 \text{ km} \) \ T=7.66 hours
Deimos: \( r = 23,500 \text{ km} \) \ T=30.4 hours

\[ \frac{T^2}{r^3} = 7.06 \times 10^{-11} \text{ hr}^2/\text{km}^3 \ (\text{phobos}) \]
\[ \frac{T^2}{r^3} = 7.12 \times 10^{-11} \text{ hr}^2/\text{km}^3 \ (\text{deimos}) \]

Since only 2 significant digits were given for the orbital radius of Phobos we can conclude that the data ARE consistent with Kepler’s Third Law

12. Use the data for Deimos in the previous problem to determine the mass of Mars. You can easily check your answer by Googling “mass of mars”.

\[ \Sigma F = ma \]
\[ F_{\text{gravity}} = \frac{mv^2}{r} \]
Gravity provides the centripetal acceleration
\[ GM_{\text{mars}} m_{\text{deimos}}/r^2 = m_{\text{deimos}} \frac{v^2}{r} \]
Deimos is the mass moving in a circle
\[ M_{\text{mars}} = \frac{v^2 r / G}{2\pi r / T} \]
Rearranging to solve for mass of mars
\[ M_{\text{mars}} = 6.4 \times 10^{23} \text{ kg} \approx 0.1 M_{\text{earth}} \]
Make sure to convert everything to SI units!

13. The free fall acceleration on the surface of Mars is 3.7 m/s/s. Determine the radius and the average density of the red planet.

\[ \Sigma F = ma \]
\[ F_{\text{gravity}} = mg \]
Gravity is only force acting in free fall
\[ GM_{\text{mars}} m_{\text{object}}/r_{\text{mars}}^2 = m_{\text{object}} g \]
r = radius of mars since that is the distance to its center
\[ r_{\text{mars}} = \sqrt{GM_{\text{mars}}/g} \]
Rearranging to solve for radius, \( g = 3.7 \text{ m/s}^2 \)
\[ r_{\text{mars}} = 3.4 \times 10^6 \text{ m} \]
Consistent with the accepted value
\[ \text{Density} = \frac{M_{\text{mars}}}{(4/3 \pi r^3)} \]
Using formula for the volume of a sphere
\[ \text{Density} = 3.9 \text{ kg/m}^3 = 3.9 \text{ g/cm}^3 \]
The density of common rocks ranges from 2.5 – 3 g/cm³
14. An exoplanet is discovered orbiting a star with a mass of $1.6 \times 10^{30}$ kg. The orbital period is 12 days ($1.04 \times 10^6$ s). What is the orbital radius of the star?

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<td>Gravity provides the centripetal acceleration</td>
</tr>
<tr>
<td>$GM_{\text{star}}m_{\text{planet}}/r^2=m_{\text{planet}}v^2/r$</td>
<td>Planet is the mass moving in a circle</td>
</tr>
<tr>
<td>$GM_{\text{star}}/r^2=\frac{4\pi^2r}{T^2}$</td>
<td>Substituting $v=2\pi r/T$</td>
</tr>
<tr>
<td>$r=(GM_{\text{star}}T^2/4\pi^2)^{1/3}$</td>
<td>Solving algebraically for $r$</td>
</tr>
<tr>
<td>$R=1.4 \times 10^{10}$ m</td>
<td>Although the star is somewhat less massive than our sun the planet orbits at less than 0.1 AU. Since radiation also follows an inverse square law this means the starlight on this planet is around 100 times more intense than on earth!</td>
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15. The Doppler Effect is used to determine that a star located $8.4 \times 10^{20}$ m from the center of a galaxy is moving at a speed of $2.6 \times 10^5$ m/s. The visible mass in that galaxy is $1.8 \times 10^{41}$ kg. What fraction of the galaxy’s mass is dark matter?

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<td>$GM_{\text{galaxy}}m_{\text{star}}/r^2=m_{\text{star}}v^2/r$</td>
<td>Star is the mass moving in a circle</td>
</tr>
<tr>
<td>$M_{\text{galaxy}}=v^2r/G=8.6 \times 10^{41}$ kg</td>
<td>Solving for mass of the galaxy* (Initially I goofed up my algebra, but my answer made no sense so I caught my mistake!)</td>
</tr>
<tr>
<td>$(8.6 - 1.8)/(8.6) = 0.79 = 79%$</td>
<td>Percentage of galaxy which is dark matter</td>
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