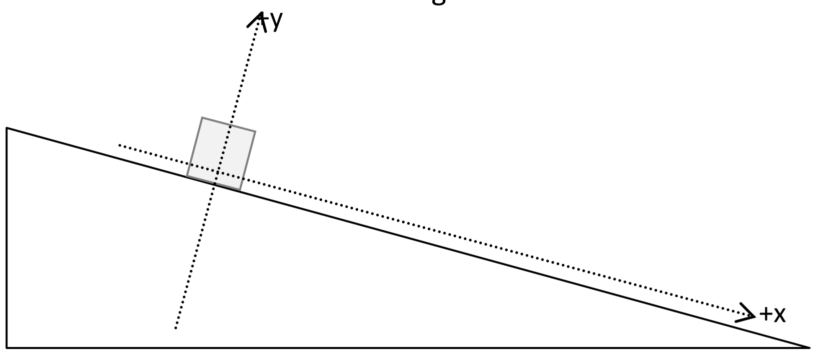
Force and Motion on an Incline

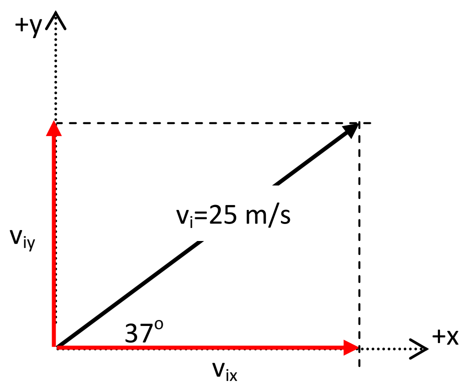
An inclined plane is basically a ramp. It is a flat surface that is sloped rather than horizontal. When solving problems about objects on an incline, it is convenient to choose a coordinate system with axes parallel and perpendicular to the surface as shown in Fig. 1.

Fig. 1



Any time we deal with forces vectors in 2-dimensions we need to resolve "off axis" or "diagonal" vectors into components. For example, when solving problems about projectiles launched at an angle we had to resolve the initial velocity into horizontal and vertical components as the example in Fig. 2 shows.

Fig. 2

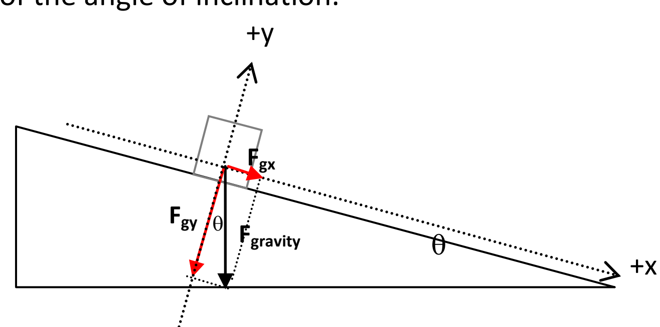


$\sin 37^\circ = v_{iy}/v_i \rightarrow v_{iy} = 25 \sin 37^\circ = 15$ m/s

$\cos 37^\circ = v_{ix}/v_i \rightarrow v_{ix} = 25 \cos 37^\circ = 20$ m/s

For incline problems, the force that will have to be broken into components in the tilted coordinate system is gravity (see Fig. 3). Thus, gravity can be thought of as doing 2 separate things. The y-component is what is pushing the object into the surface. It equals the weight of the object times the cosine of the angle of the incline and is usually canceled by the normal force pushing outwards. The x-component of gravity is the force that is trying to accelerate the object down the ramp. It equals the weight times the sine of the angle of inclination.

Fig. 3



$F_{gy} = F_g \cos\theta$

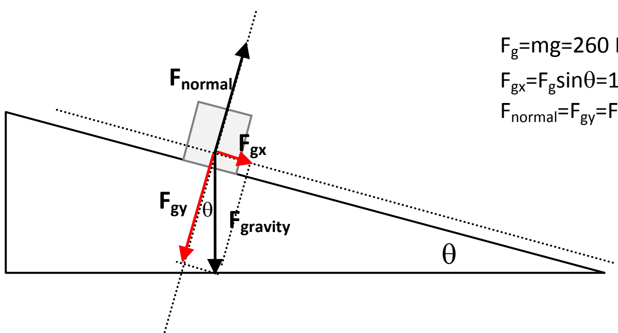
$F_{gx} = F_g \sin\theta$

One trick that makes it easier for some students to solve these problems is to rotate the free-body diagram so that it more closely resembles the horizontal motion problems we have already practiced. You can see what this looks like in the examples that follow.

Here are 2 worked examples to demonstrate how to go about solving inclined plane problems.

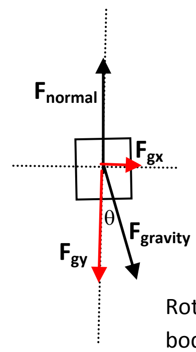
<u>Ex. 1 No Friction</u>

At what rate will a 26 kg box accelerate if slides down a frictionless slope that is inclined at 22.6^{o} above horizontal?



F_g=mg=260 N
F_{gx}=$F_g$sinθ=100 N
F_{normal}=F_{gy}=$F_g$cosθ=240

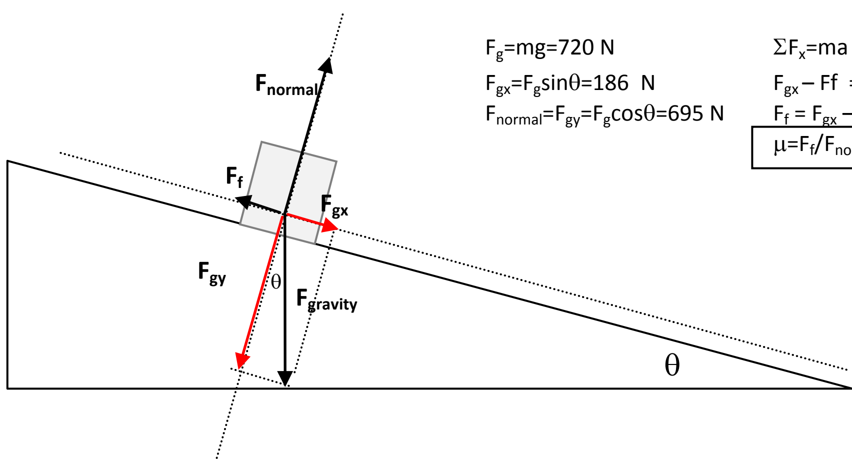
ΣF_x=ma
F_{gx}=ma
a=100/26=3.8 m/s^2



Rotated free body diagram

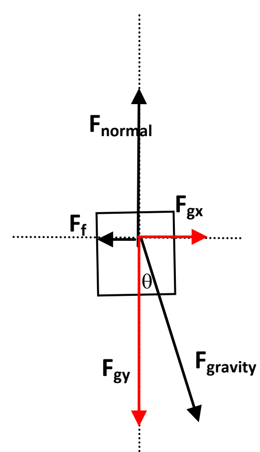
<u>Ex. 2 With Friction</u>

A 72 kg box sliding down a 15^{o} incline accelerates at a rate of 0.70 m/s^2. What is the coefficient of friction?



F_g=mg=720 N
F_{gx}=$F_g$sinθ=186 N
F_{normal}=F_{gy}=$F_g$cosθ=695 N

ΣF_x=ma
F_{gx} – Ff = ma
F_f = F_{gx} – ma = 135.6 N
μ=F_f/F_{normal}=0.20



Rotated free body diagram

INCLINED PLANES

No friction

1. $m=50$ kg $\theta=37^\circ$ $F_{gx}=?$ $F_{gy}=?$ $a=?$
2. $m=230$ kg $\theta=60^\circ$ $F_{gx}=?$ $F_{gy}=?$ $a=?$
3. $F_g=300$ N $\theta=30^\circ$ $F_{gx}=?$ $F_{gy}=?$ $a=?$

With friction

4. $m=10$ kg $\theta=53^\circ$ $F_f=20$ N $F_{gx}=?$ $F_{gy}=?$ $a=?$ $\mu=?$
5. $m=13$ kg $\theta=67.4^\circ$ $F_f=25$ N $F_{gx}=?$ $F_{gy}=?$ $a=?$ $\mu=?$
6. $m=14.1$ kg $\theta=45^\circ$ $\mu=0.20$ $F_{gx}=?$ $F_N=?$ $F_f=?$ $a=?$

Word problems

7. A 75 kg skier accelerates from rest to a speed of 12 m/s in 3.0 seconds on a frictionless hill. What is the slope of the hill?
8. A 46 g domino slides down a 30° incline at a constant speed. What is the coefficient of friction?
9. A boy and his sled have a combined mass of 65 kg. What is their acceleration as they start down an icy 22.6° incline with a coefficient of friction equal to 0.10?
10. The boy in the previous problem is pulled back to the top of the hill at a constant speed by a tow rope. What is the tension in the rope?